Joint Power and Backhaul Bits Allocation for Coordinated Multi-Point Transmission

Hazem M. Soliman, Ahmed A.Naguib, Omar A. Nasr, Mohamed M. Khairy and Khaled Elsayed
EECE Department, Cairo University

***Abstract*—In this paper, we study the problem of backhaul sharing in Network MIMO. In case of a backhaul of limited number of bits, we investigate how to allocate these bits jointly with the base-stations power among users in order to maximize the sum-rate under fairness constraints. Firstly, we study this problem with the assumption of one user per cell. We find that binary allocation of power is optimal. Then, the multi-user case is considered. A solution of the optimization problem is developed under high signal-to-interference and noise ratio(SINR). We prove that under these assumptions, backhaul bits allocation among the cells or among the users of each cell has no effect on the sum-rate of system. Hence, allocating these bits in order to achieve fairness conditions is possible without any loss in the sum-rate.**

***Index Terms*—Multi-cell, backhaul sharing, power allocation**

I. Introduction

Interference management is one of the key challenges facing future wireless communication systems. The conventional technique to deal with interference is to limit the re-usability of resources (time, frequency, code,...) to introduce some kind of orthogonality between users. The more recent approach attempts to make use of interference or at least coordinate it through the use of cooperative transmission [1]. Multi-Cell MIMO or Network MIMO is a new technology for cellular base-stations that mitigates interference by coordinating base-stations transmission. 3GPP LTE-A and IEEE 802.16m have recently chosen Network MIMO as a means to increase the cell-edge and system throughput [2] [3].

A fundamental challenge in Multi-Cell MIMO networks is the issue of limited backhaul bandwidth. Multi-Cell processing or full co-operation among base-stations requires the exchange of full channel state information (CSI) and data transmitted to users among all base-stations, this requires a very high-speed backhaul. Another type of collaboration called Interference Coordination [1] requires the exchange of CSI only to perform some form of coordinated beamforming. Several attempts to reduce backhaul requirements through distributed cooperation, statistical CSI exchange or clustered cooperation have been proposed [4] [5] [6].

The installation of Network MIMO based base-stations requires new resources in the network. One type of these resources is the backhaul bandwidth. Two types of resources are considered in this paper, Power allocation and backhaul bits allocation. The problem of power allocation for two cells without cooperation was studied in [7]. The authors sowed that the solution is in fact binary with the each base-station either operating at maximum or zero power. While the problem of backhaul sharing was studied in [8], the authors showed that for a high SNR, the sum-rate depends only the total backhaul rather than the users shares, and therefore proposed two fair allocations of the backhaul bandwidth.

This work was funded by the Egyptian National Telecommunications Regulatory Authority (NTRA).

We divide the problem of backhaul sharing into two parts. First, we consider two cooperating base-stations with each base-station serving one user. We study how to optimally allocate power and backhaul bandwidth between the two base-stations. Our solution show that binary allocation of power and backhaul is optimal. However, this allocation depends on the total bandwidth of the backhaul. The optimal allocation of the backhaul bits is also developed. Second, we consider the multi-user case in which N users are served by each base-station. We study this problem in the high SNR regimes, leaving the general case for future work. We proved that the sum-rate doesn't depend neither on the allocation of the backhaul bits among base-station nor among the users of each cell. We show we can achieve better fairness for the same sum-rate by clever allocation of the backhaul bandwidth. We also find the optimum power allocation among the users of each cell assuming all base-stations have the same maximum power.

The rest of the paper is organized as follows, Section II introduces the system model. The one user case is studied in Section III. The multi-user case is studied in Section IV. Section V shows the simulation results. Finally section VI concludes the paper.

II. System Model

We consider a Wyner type [9], two base stations, N-user per cell MIMO downlink system. Each base station has $M = 2N$ antennas, while users are each equipped with a single antenna. Channel is taken from the Zero-mean Circularly-symmetric Complex-Gaussian model $(ZMCSCG)$[10]. We also use the simple yet efficient Zero-Forcing precoder, its columns are normalized to obey maximum power limit condition. Hence the received signal may be expressed as

$$\mathbf{y} = \mathbf{L}_1\mathbf{H}_1\mathbf{W}_1\mathbf{x}_1 + \mathbf{L}_2\mathbf{H}_2\mathbf{W}_2\mathbf{x}_2 + \mathbf{n} \tag{1}$$

where $\mathbf{y}$ is the received signal vector, $\mathbf{x}_k$ is the vector carrying symbols to be transmitted from base-station k, $\mathbf{H}_k$ is the channel matrix between users and base-station k, $\mathbf{W}_k$ is the linear beamformer used at base-station k, $\mathbf{L}_k$ is a diagonal matrix with diagonal elements representing the square root of the power path-loss, $(l_{ii,k})^2 = K\left(\frac{d_{k,i}}{d_o}\right)^{-\beta}$, where $d_{k,i}$ is the distance between base-station k and user i, and K is the loss at a reference distance d_o [11]. $\mathbf{n}$ is the noise vector. Without loss of generality, we will focus on users in the first cell. For a user in the first cell, the first term represents the useful signal as well as the same cell interference, the second represents the other cell interference and the third is the noise term. Therefore the instantaneous received signal to interference and noise ratio $(SINR_i^t)$ for user i in the first cell is

$$SINR_i^t = \frac{Pt_{1,i}\,(l_{ii,1})^2\left|\mathbf{h}_{1,i}^H\mathbf{w}_{1,i}\right|^2}{\sigma^2 + \sum\limits_{1<j<N} Pt_{2,j}\,(l_{ii,2})^2\left|\mathbf{h}_{2,i}^H\mathbf{w}_{2,j}\right|^2} \tag{2}$$

where $\mathbf{h}_{k,i}$ is the channel vector between base-station k and user i, $\mathbf{w}_{k,i}$ is the beamforming vector between base-station k and user i, and $Pt_{k,g}$ is the symbol power transmitted from base station k intended to user g and σ^2 is the noise power which is assumed to be the same for all users. Let $P_{k,i,g}$ denote the received power at user i due to the transmitted power $Pt_{k,g}$, $P_{k,i,g} = Pt_{k,g} \times (l_{ii,k})^2$. [1] Then $(SINR_i^t)$ is written as

$$SINR_i^t = \frac{P_{1,i,i}\left|\mathbf{h}_{1,i}^H\mathbf{w}_{1,i}\right|^2}{\sigma^2 + \sum\limits_{1<j<N} P_{2,i,j}\left|\mathbf{h}_{2,i}^H\mathbf{w}_{2,j}\right|^2} \tag{3}$$

Each base station is assumed to have perfect knowledge of its own users channels. These channels should then be conveyed through the backhaul to the other base station to do coordinated beamforming. However, due to limited backhaul bandwidth, only quantized versions of the channels may be exchanged between the coordinating base-stations. Each base station uses the perfectly known channels of its own users, and the quantized channels of the other cell users to design a beamforming matrix $\mathbf{W}$. Even with coordinated Zero-forcing beamforming, the users whose channels were quantized will suffer from interference due to quantization.

An analytical formulation for the average received $SINR_i$ using a beamformer based on a uniformly quantized-channel was found in [8]. It should be noted that the interference from the same cell users equals zero due to using zero-forcing beamforming based on perfectly known channels. The received multi-cell interference I_i at user i in the first cell is

$$\mathbf{E}\{I_{qi}\} = \sum_{1<j<N} P_{2,i,j} \times Q_i \tag{4}$$

where Q_i is the quantization noise and is given by [12]

$$Q_i = \mathbf{E}\{\left|nq_{i,k}\right|^2\} = \frac{\text{const}}{2^{-2\times b_i}} \tag{5}$$

where b_i is the number of quantization bits per channel given to user i and it represents the constraint in our optimization problem such that $\sum\limits_{1<i<N} b_i = D$ where D is the total number of available quantization bits in the backhaul.

[1]The same cell interference is considered zero in this expression due to perfect ZF for the same cell

The average $SINR_i$ may be written as

$$SINR_i = \frac{P_{1,i,i}}{\sigma^2 + \sum\limits_{1<j<N} P_{2,i,j} \times Q_i} \tag{6}$$

Assuming each user has a rate r_i (in bits/symbol/Hz), the sum-rate for N-users in each cell is given by

$$\sum_{1<i<N} r_i = \sum_{1<i<N} \log_2\left(1 + SINR_i\right) \tag{7}$$

Although the average $SINR_i$ is independent of the channel, the values of $P_{1,i}$ and $P_{2,i}$ vary greatly according to the user position due to the path loss.

III. Two cells with one user per cell

As we mentioned in Section I, we will study the problem of backhaul sharing jointly with base-stations power allocation in two cases. Firstly, we will consider the setting of two cooperating base-stations with only one user served by each one. In this setting, the power of each base-station is constrained by a maximum power limit P_{max}. We want to find the joint optimal allocation of power and backhaul bits in order to maximize the users sum-rate.

The two-cell sum-rate is defined as:

$$R_{sum} = \sum_{\substack{\forall i,j\in\{1,2\}\\ i\neq j}} \log_2\left(1 + \frac{P_1 \times l_{i,i}}{\sigma^2 + P_2 \times l_{i,j} \times Q_i}\right) \tag{8}$$

The objective now is to find Q_1, Q_2, P_1, P_2 to maximize the previous function subject to the following constraints:

$$\begin{aligned} Q_1 \times Q_2 =& \frac{\text{const}}{2^{-2\times(b_1+b_2)}} = \frac{\text{const}}{2^{-2\times D}} = Q_{min} \\ & 0 < P_1 < P_{max} \\ & 0 < P_2 < P_{max} \\ & Q_1 < 1, Q_2 < 1 \end{aligned} \tag{9}$$

where P_{max} is the maximum power to be transmitted from either base station.

Substituting Q_2 with Q_{min}/Q_1, the Lagrangian of the above problem can be written as:

$$\begin{aligned} \mathbf{J} =& \log_2\left(1 + \frac{P_1 \times l_{1,1}}{\sigma^2 + P_2 \times l_{1,2} \times Q_1}\right) \\ &+ \log_2\left(1 + \frac{P_2 \times l_{2,2}}{\sigma^2 + P_1 \times l_{2,1}\,(Q_{min}/Q1)}\right) \\ &+ \mu_1\,(P_1) + \mu_2\,(P_1 - P_{max}) + \mu_3\,(P_2) \\ &+ \mu_4\,(P_2 - P_{max}) + \mu_5\,(Q_1 - 1) \end{aligned} \tag{10}$$

The problem of power allocation was studied in [7]. The authors considered two non-cooperating base-stations serving one user per cell where the sum rate was dependant on the channels gains. The authors showed the optimum power allocation is in fact binary, that is each base-station is either operating at maximum or zero power depending on the channel gains. In our context we assume coordinated ZF, however due to the quantization effect we have SINR expressions analogous to those used in [7].

A. Users at equal distances (d) from their base-stations

In this case, $l_{1,1} = l_{2,2}$, $l_{1,2} = l_{2,1}$. Hence, the problem turns to be similar to that in scenario(2) of [7]. Using similar analysis, It can be shown that power allocation that maximizes sum-rate in this case is binary power allocation, i.e., the maximum sum-rate is achieved if P_1 and P_2 are eitherP_{max}, P_{max} or $P_{max}, 0$. This allocation is independent of Q_1 and Q_2, it depends only on the their product, i.e., Q_{min} as follows:

$$P_1, P_2 = \begin{cases} P_{max}, P_{max} & \text{if } Q_{min} < Q_{critical} \\ P_{max}, 0 & \text{if } Q_{min} > Q_{critical} \end{cases}$$

where

$$Q_{critical} = \frac{1+\gamma'}{\gamma'^2} \times \left(\frac{2d_o - d}{d}\right)^{2\beta} \gamma' = \frac{P_{max} \times k \times \left(\frac{d}{d_o}\right)^{-\beta}}{\sigma^2} \tag{11}$$

From [7], the power mode switching is controlled by a constant that equals in our case $Q_1 \times Q_2 = Q_{min}$. Since Q_{min} is just a constant not an optimization parameter, therefore the solution for the power modes is the same in our context. Therefore the total quantization noise due to limited backhaul bits, Q_{min} controls the optimal power mode. This result can be simply justified as follows: If the backhaul capacity is small, high Q_{min}, the interference of one base station on the user of the other cell will be high, and hence it is better to shut this base-station off. In contrast to that, for large backhaul capacity, allowing the two base stations to transmit with maximum power will achieve higher rate as the interference caused by one base-station on the user of the other cell will be small.

From the previous discussion, it is illustrated that our problem of joint power and backhaul bits allocation is found to be separable. For the power mode, $\{P_{max}, o\}$, the $Q's$ are removed from the rate equation, therefore it is not an optimization parameter anymore. However, for the second power mode, $\{P_{max}, P_{max}\}$, we can solve the Lagrangian and the solution is given by either $Q_1 = Q_2 = \sqrt{Q_{max}}$ or $Q_1 = 1 \& Q_2 = Q_{max}$. It can also be proved that the switching point between the two solutions is the same switching point between the two power modes. Therefore the solution set is given by

$$\begin{cases} \left(P_{max}, \sqrt{Q}_{min}\right), \left(P_{max}, \sqrt{Q}_{min}\right) & \text{if } Q_{min} < Q_{critical} \\ \left(P_{max}, Q_{min}\right), (0, 1) & \text{if } Q_{min} > Q_{critical} \end{cases} \tag{12}$$

Note that however in the second case, the $Q's$ are out of consideration, therefore the only solution of interest is $\left\{\sqrt{Q_{min}}, \sqrt{Q_{min}}\right\}$

B. Users at Unequal distances from base-stations

In this case, $l_{1,1} \neq l_{2,2}$, $l_{1,2} \neq l_{2,1}$. Unlike the previous parts, binary allocation of the backhaul is not always optimal, and we have to solve the problem in (10). However, to simplify the solution, we make use of the results of the previous case. Assuming $d_1 < d_2$, where d_i is the distance between user i and base-station i. Then we can note that the users sum-rate in case of different distances is also upper bounded by the sum-rate of equal distance $d = d_1$. Hence the solution of the problem can be divided into two regions.

If the power allocation that achieves the maximum sum-rate in case of equal distances $d = d_1$ is switching one of the base-stations off, then, this will be also the optimum power allocation for the case of different distances to base-stations.

In the other case, when the optimum power allocation in case of equal distances leads to switching the two base-stations on, then, this will be also the power allocation that achieves the maximum sum-rate, but, the optimization problem in(10) will lead to Q_1, Q_2 that are not binary in general. These results are summarized in the following equations:

$$(P_1, P_2) = \begin{cases} (P_{max}, P_{max}) & \text{if } Q_{min} < Q_{critical(d_1)} \\ (P_{max}, 0) & \text{if } Q_{min} > Q_{critical(d_1)} \end{cases}$$

where $Q_{critical}(d_1)$ is that defined in (11) but with $d = d_1$ which is the minimum distance.

Now, in case of the first power allocation (12), as d_2 increases, switching the two base-stations on will be the optimum power allocation until a certain threshold d_{th} after which switching P_2 off will be optimum. This threshold distance is obtained from the following condition:

$$\sum_{i,j=\{1,2\}, i\neq j} \log_2\left(1 + \frac{P_1 \times l_{i,i}}{\sigma^2 + P_2 \times l_{i,j} \times Q_i^*}\right) = \log_2\left(1 + \frac{P_1 \times l_{1,1}}{\sigma^2}\right) \tag{13}$$

At high SNR, the threshold distance which is expressed implicitly in the previous equation, can be found to be:

$$d_{th} = 2\left(\left(\frac{Q_2^*\left(1 + P_{max}/\sigma^2\right)}{1 + \frac{P_{max}K\left(\frac{d_1}{d_o}\right)^{-\beta}}{\sigma^2 + P_{max}K\left(\frac{2-d_1}{d_o}\right)^{-\beta} Q_1^*}} - Q_2^*\right)^{\beta} + 1\right)^{-1} \tag{14}$$

where Q_1^*, Q_2^* is the optimum Q_1 and Q_2 in case of switching the two base-stations on, and is given by solving the Lagrangian problem (10) but fixing $P_1 = P_2 = P_{max}$. The solution of this problem can be simply derived to be the positive real root of the following quadratic equation:

$$\begin{aligned} & Q_1^{*^2}\left(C\left(1 + \gamma\, l_{2,2}\right) - \left(\gamma\, l_{1,2}\right)^2\right) \\ & + Q_1^*\left(C\gamma\, l_{2,1} Q_{min}\left(2 + \gamma\, l_{2,2}\right) - \gamma\, l_{1,2}\left(2 + \gamma\, l_{1,1}\right)\right) \\ & + C\left(\gamma\, l_{2,1} Q_{min}\right)^2 - \left(1 + \gamma\, l_{1,1}\right) = 0 \end{aligned} \tag{15}$$

where

$$C = \frac{1}{Q_{min}}\left(\frac{d_1\left(2d_o - d_1\right)}{d_2\left(2d_o - d_2\right)}\right)^{-\beta}, \gamma = \frac{P_{max}}{\sigma^2} \tag{16}$$

IV. Multi-user cells

This section is divided into two subsections. Firstly, we will study the optimal joint allocation of power and backhaul bits among users to maximize the system sum-rate. In this subsection, we will assume that each cell has a distinct backhaul link that can't be shared with the other base-station. In the second part, fairness algorithms will be applied to our problem to study their effect on the sum-rate.

A. Joint power and backhaul bits allocation

The sum-rate can be defined as:

$$\sum_{1<i<N} r_i = \sum_{1<i<N} \log_2 (1 + SINR_i) \tag{17}$$

and for large SINR, we approximate rates by $r_i = \log_2 (SINR_i)$, therefore we may write sum-rate as

$$\sum_{1<i<N} r_i = \sum_{1<i<N} \log_2 (SINR_i) \tag{18}$$

Also for a low-to-moderate number of quantization bits which makes $\sigma^2 \ll I_{qi}$, and using (6), we can write:

$$\sum_{1<i<N} r_i = \log_2 \left(\frac{\prod\limits_{1<i<N} P_{1,i,i}}{\prod\limits_{1<i<N} \left(\sum\limits_{1<j<N} P_{2,i,j} \times Q_i \right)} \right) \tag{19}$$

and from (5)

$$\sum_{1<i<N} r_i = \log_2 \left(\frac{\prod\limits_{1<i<N} P_{1,i,i}}{\prod\limits_{1<i<N} (l_{i,2}) P_{max} \times C \times 2^{-2D}} \right) \tag{20}$$

where P_{max} is the total max power of the base-station, C is the constant in (5) and D is the total number of quantization bits and equals $\sum\limits_{1<i<N} b_i$. Therefore, as long as $\sigma^2 \ll I_{qi}$, the sum-rate is dependant only on the total number of quantization bits D rather than its distribution.

Consequently, the joint power and backhaul bits allocation problem is simplified to a power allocation problem. For two cells, it can be shown using (22) that the problem is to find the allocation of power among users that maximizes:

$$\prod_{1<i<N} Pt_{1,i,i} \tag{21}$$

where, $Pt_{1,i}$ is the power allocated to user i in cell 1 by base-station 1. The solution of this problem is obviously allocating equal transmitted power to all users in the cell.

B. Fairness optimization

As the sum-rate doesn't depend on the allocation of backhaul bits, this problem is particulary suitable for fairness optimization. In [8] we proposed two schemes to share backhaul, Equal SIR and Equal Interference. To study the performance of these schemes, we compare them with the following numerically solved optimization problems.:

1) Max Min SINR: we may formulate the problem as

$$\begin{aligned} \max_{b_i} \min_{i} \quad & SINR_i \\ \text{s.t.} \quad & \sum_{1<i<N} b_i = D \end{aligned} \tag{22}$$

here we aim to maximize the minimum $SINR$ subject to a limited total number of quantization bits. This problem is useful for finding the maximum possible minimum rate and comparing it with the minimum rate found using our schemes.

2) Max Min SINR with mean rate condition: we may formulate the problem as

$$\begin{aligned} \max_{b_i} \min_{i} \quad & SINR_i \\ \text{s.t.} \quad & \sum_{1<i<N} b_i = D \\ & \sum_{1<i<N} r_i \geq R \end{aligned} \tag{23}$$

here we aim to maximize the minimum $SINR$ subject to a limited total number of quantization bits and that the resulting sum-rate is greater than a certain value R. Here we put another constraint on the above problem. We want to achieve the best fairness, the maximum possible minimum rate, without completely sacrificing the mean rate performance.

3) Max Sum-rate with minimum condition: we may formulate the problem as

$$\begin{aligned} \max_{b_i} \quad & \sum_{1<i<N} r_i \\ \text{s.t.} \quad & \sum_{1<i<N} b_i = R \\ & min \{r_i\} \geq r_{min} \end{aligned} \tag{24}$$

here we aim to maximize the sum of the rates r_i subject to a limited total number of quantization bits and that the resulting minimum rate is greater than or equal to a certain value r_{min}. This problem is used to maximize the mean rate performance while still satisfying a certain fairness performance represented by the minimum rate condition.

V. Simulation Results

For the one-user per cell case with equal distances to base-stations, simulation is done for $\gamma = \{5, 50\}$, $P_{max} = 100 mWatt$. Figure (1) shows how the optimal modes, both power and backahul, switch at $Q_{critical}$. Figure (2) shows the sum-rate versus normalized distance for several numbers of quantization bits for the unequal distance case. It can be seen from the figure that the optimal power mode switches at a certain $Q_{critical}$ that depends on the users distance and their γ's. This threshold, $Q_{critical}$, switches the power mode between $\{P_{max}, P_{max}\}$ and $\{P_{max}, 0\}$. It also switches the backhaul between $\left\{\sqrt{Q_{max}}, \sqrt{Q_{max}}\right\}$ and $\{Q_{max}, 1\}$ in the equal distance case.

For the multi-user case, The simulation is done for a two-base stations, 4 users per cell. Backhaul bandwidth is increased from 1 to 40 bits on average per user. The reference

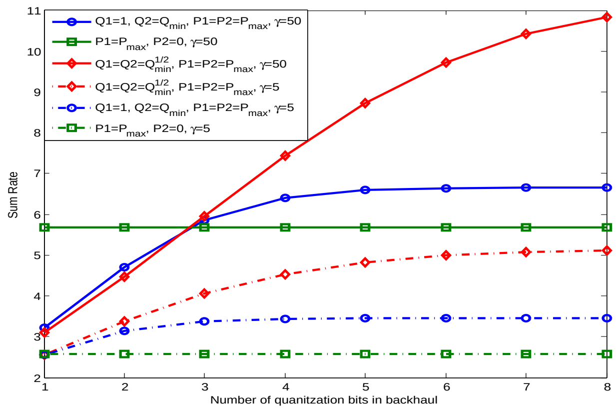

Fig. 1. Sum Rate versus number of quantization bits for the equal distance case

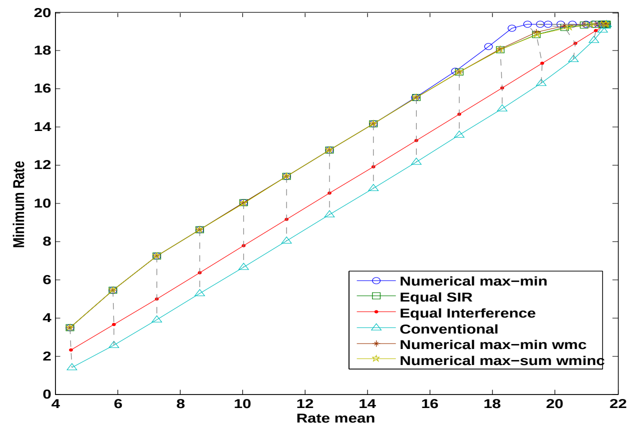

Fig. 3. Minimum rate versus rate mean

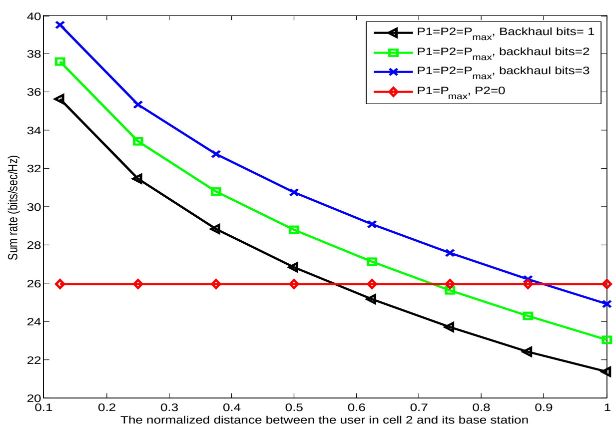

Fig. 2. Sum Rate versus distance for the unequal distance case

distance d_o is assumed to be 1600m and is also considered the cell radius. Path loss exponent is 3.8. Users per cell are allocated every 400m starting from 400m to 1600m. Base station power is equal to 10 watt. Performance is compared with the conventional scheme that equally allocates bits among users.

In figure (3) we plot the minimum rate versus mean rate for several schemes. Here we plot the mean found from solving problem (IV-B2) where the constant mean is that of the Equal SIR scheme. We also plot the mean of problem (IV-B3) where the constant minimum is that of Equal SIR scheme. We may notice from the figure that the maximum minimum solution comes at the expense of reducing mean rate unlike the Equal SIR scheme. We also notice that trying to optimize the Equal SIR scheme by trying to increase its mean rate or its minimum rate yields negligible improvement, in other words Equal SIR may achieve optimal trade-off between mean and minimum.

VI. Conclusion

In this paper, we studied the problem of how to best allocate the backhaul bandwidth and base-stations power jointly among users in a multi-cell MIMO system in order to maximize the sum-rate under fairness constraints. We solved this problem with general conditions in the case of one user per cell, and in case of multi-users cells but with the assumption of high SINR. We showed that as the sum-rate is independent on the backhaul bandwidth allocation, the problem of joint power and backhaul bits allocation can be separated. This also facilitates achieving fairness conditions for the different users.

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