Interference Alignment Performance on MIMO X Channels with Imperfect Channel Knowledge

AbdulRahman Mahmoud[◇], *Mostafa El-Khamy*[◇] *and Khaled Elsayed*[∇]
[◇]Electrical Engineering Department, Faculty of Eng., Alexandria University, Alexandria 21544, Egypt
[∇]Department of Electronics and Communications Engineering, Cairo University, Giza 12613, Egypt
abdulrahman@alexu.edu.eg, m_elkhamy@ieee.org, khaled@ieee.org

***Abstract*—Interference alignment (IA) can achieve the optimal degrees of freedom in interference-limited wireless systems. Popular IA schemes assume perfect global knowledge of the MIMO channels at all transmitters and receivers. In this paper, we investigate the error rate performance of interference alignment in MIMO $M \times 2$ X systems at practical signal to noise ratios. Through theoretical analysis and Monte Carlo simulations, we analyze the effect of imperfect channel estimation at the receiver and transmitter on the bit error rate performance of zero-forcing interference alignment.**

***Index Terms*—interference alignment, X-channel, degrees of freedom, MIMO, zero-forcing, channel estimation error, sum rate**

I. Introduction

Interference alignment (IA) can increase the bit-rate of communication systems in interference-limited regimes. IA schemes aim to align interfering signals such that they span a limited subspace of the received signal space, thus leaving the rest of the signal space free from interference. IA schemes can be classified as closed-form algorithms (e.g. [1], [2], [3]) or iterative algorithms (e.g. [4], [5], [6]). Closed form IA schemes can achieve perfect IA without any iterations but require global channel knowledge which is utilized in determining vectors used in transmitter precoding and receiver decoding. In iterative IA schemes, assuming constant channels, users iteratively update their transmit precoding vectors and receive encoding vectors in order to minimize the total interference or maximize their signal-to-interference ratio [4], [5], [6].

In an $M \times N$ X-channel, there are M transmitters and N receivers, where each transmitter sends an independent message to each receiver at each time slot, forming MN Tx-Rx pairs. When all transmitters and receivers have single antennas, it was shown that a DOF of $\frac{MN}{M+N-1}$ (i.e. MN total DOF over $M + N - 1$ channel extensions) can be asymptotically achieved [7]. The same DOF can be achieved by multiple antennas instead of channel extensions. For a general number of users in the X-channel, only partial IA can be achieved. In a MIMO $M \times 2$ X-channel, when all transmitters and receivers have $M + 1$ antennas, perfect IA can achieve the optimal DOF of $2M$ [3]. Since perfect IA schemes are designed assuming perfect knowledge of channel state information (CSI) at the transmitters and receivers, their performance at finite SNRs will be dependent on the accuracy of the available channel estimates.

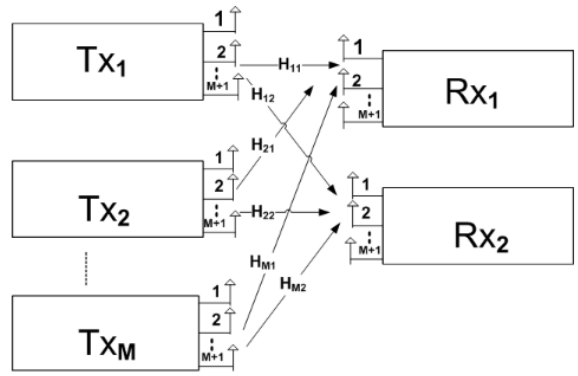

Fig. 1. System model of MIMO $M \times 2$ X channel

In this paper, we analyze the bit error rate (BER) performance of IA at finite SNRs, where we consider the perfect IA scheme in MIMO $M \times 2$ X channels, with $M+1$ antennas at each node, and receivers deploying zero-forcing detectors [3] [7]. The effect of uncertainty in CSI have been previously considered for MIMO single user systems with zero-forcing receivers [8] and for the iterative IA scheme of [6], [9]. Since perfect $M \times 2$ IA schemes requires global channel knowledge at the receivers as well as at the transmitters, we investigate the effect of imperfect CE on the bit error rate performance when the CE error is present at transmitters only or at receivers only or in the more practical case of CE error at both transmitters and receivers. We also consider the case when only a subset of the M transmitters are active. We will show theoretical formulas for the performance on Rayleigh channels and verify them by Monte Carlo simulations. We also investigate the effect of CE on the sum rate.

The rest of the paper is organized as follows: In section II, we present the IA signaling scheme used for the X-channel and the system model. In section III, we analyze the bit error rate performance of the the $M \times 2$ IA system with zero-forcing detectors and compare it with the single user system when perfect global CSI is available. We analyze the effect of CE error on the performance when the CE error is at the receiver only (section IV), or at the transmitter only (section V) or at both the transmitter and receiver (section VI).In section VII, we conclude the paper.

II. System Model

We consider the system model of $M\times 2$ wireless X-channel with M transmitters and 2 receivers as shown in Fig. 1. Each transmitting or receiving node has $M+1$ antennas. At any time slot, each transmitter i, Tx_i, is sending the independent data symbols x_{i1} and x_{i2} to the two receivers Rx_1 and Rx_2, respectively. The modulated data symbols are i.i.d, drawn uniformly from a constellation of size K, and $\mathrm{E}[x_{ij}x_{ij}^H] = E_s$, where $\mathrm{E}[X]$ is the expectation of X and X^H is the complex conjugate transpose of X. H_{ij} is the $(M+1)\times(M+1)$ independent complex channel gain matrix between Tx_i and Rx_j and its elements are i.i.d complex Gaussian with zero mean and unit variance. The complex additive white Gaussian noise (AWGN) vector at input of receiver Rx_i is independently given by $\mathbf{n_i} \sim CN(0, N_0\mathbf{I})$, where $\mathbf{I}$ is the $(M+1)$ identity matrix.

We deploy the IA scheme of [1], [3], where the interference will be aligned at two randomly chosen directions I_1 and I_2 in the signal space at receivers Rx_1 and Rx_2, respectively. The vectors I_1 and I_2 are assumed to be independent, Rayleigh distributed and of unit norm. Each transmitted symbol x_{ij} is precoded using a precoding matrix T_{ij} such that all the interference received at Rx_1 and Rx_2 is aligned at directions I_1 and I_2, respectively. The transmitted vector from Tx_i is

$$X_i = T_{i1}x_{i1} + T_{i2}x_{i2}, \tag{1}$$

where $T_{i1} = H_{i2}^{-1}I_2$ and $T_{i2} = H_{i1}^{-1}I_1$ require CSI knowledge at the transmitters. The received signal at Rx_j (for $j=1,2$) is

$$\mathbf{y_j} = \sum_{i=1}^{M} H_{ij}X_i + \mathbf{n_j} = B_j\mathbf{x_j} + \mathbf{n_j}, \tag{2}$$

where the $(M+1)\times(M+1)$ matrix B_j and the $(M+1)$ column vector $\mathbf{x_j}$ are given by

$$B_j = \left[\begin{array}{cccccc} H_{1j}T_{1j} & H_{2j}T_{2j} & \cdots & H_{Mj}T_{Mj} & I_j \end{array}\right], \tag{3}$$

$$\mathbf{x_j} = \left[\begin{array}{ccccc} x_{1j} & x_{2j} & \cdots & x_{Mj} & \sum_{i=1,k\neq j}^{M} x_{ik} \end{array}\right]^T, \tag{4}$$

where $[.]^T$ denotes matrix transpose. From (2), the zero-forcing (ZF) solution to retrieve the transmitted symbols at Rx_j, $(j=1,2)$ is

$$\mathbf{z_j} = B_j^{-1}\mathbf{y_j} = \mathbf{x_j} + B_j^{-1}\mathbf{n_j}, \tag{5}$$

so the first M components of $\mathbf{z_j}$ are the interference free noisy estimates of the M symbols transmitted to Rx_j and the last component represents the aligned interference symbols. Since B_j is almost surely non-singular for randomly chosen I_1 and I_2, $2M$ degrees of freedom (DOF) can be achieved using this perfect IA scheme [3].

The channel matrix H_{ij} is estimated at the transmitters or receivers as $\widehat{H}_{ij}$. Assuming the normalized mean square error (NMSE) between any gain element in H_{ij} and its corresponding estimate in $\widehat{H}_{ij}$ is equal to e^2, $\widehat{H}_{ij}$ can be expressed as [10], [8]

$$\widehat{H}_{ij} = H_{ij} + e\Omega_{ij}. \tag{6}$$

The CE error $e\Omega_{ij}$ is independent of H_{ij} and the elements of Ω_{ij} are assumed to be i.i.d complex Gaussian of zero mean and unit variance such that $\mathrm{E}[\Omega_{ij}\Omega_{ij}^H] = (M+1)\mathbf{I}$.

In the following sections, we theoretically analyze the error rate performance of the $M\times 2$ IA system for the different cases of perfect or imperfect CE available at the transmitters or receivers, when the ZF receiver of (5) is used.

III. Analysis of X-channel IA with Perfect CSI

We analyze the BER of perfect IA, when perfect global CSI is available at all transmitters and receivers. We show analytical results and validate them using simulations.

A. Analytical Performance

For the $M\times 2$ MIMO X-channel IA system, the expected bit error rate (BER) over the $2M$ transmitted symbols is equal to the average BER over the M symbols received at Rx_1 or Rx_2 due to symmetry. Moreover, since all channel gains are i.i.d and all data symbols are i.i.d drawn from a uniform distribution, then the expected BER of Rx_1 is equal to the expected BER of Rx_1 due to data symbols x_{ij} transmitted from Tx_i. Thus, the expected system BER performance is equivalent to the expected BER performance of the link between Tx_i and Rx_j.

When global CSI is available at all transmitters and receivers with no error ($e=0$), by (5) the noisy estimate of x_{11} is z_{11}(the first element of $\mathbf{z_1}$). By IA construction, this ZF solution is free from interference with symbols intended to other receivers. Thus, the expected BER of the $M\times 2$ IA system is equivalent to the average BER of the single user 1×1 MIMO system with a zero-forcing detector. Closed form formulas that tightly approximate the BER performance of K-QAM and K-PSK modulation (QAM and PSK modulation with K constellation points) in an AWGN channel where presented in [11] and [12], respectively. These approximations where extended to single user MIMO system with N_t transmit antennas and N_r receive antennas, by averaging them over the SNR per symbol which follows a chi-square distribution with $2(N_r - N_t + 1)$ degrees of freedom [8]. Then, for the special case of $N_t = N_r = M+1$ antennas, the BER performance of the $M\times 2$ IA system using K-PSK modulation can be approximated as [1]

$$BER_{K-\mathrm{PSK}} \approx \frac{1}{\max(\log_2 K, 2)} \sum_{i=1}^{\min(2,\lceil K/4\rceil)} (1-\mu_i)$$
$$\text{where } \mu_i = \sqrt{\frac{\gamma_s \sin^2\left((2i-1)\pi/K\right)}{1+\gamma_s \sin^2\left((2i-1)\pi/K\right)}}, \tag{7}$$

[1]For QAM constellations, approximated BER expressions as a function of γ_s exist, but we only consider PSK modulations due to space limitations.

and γ_s is the effective symbol SNR gain. For the special case of no CE error then

$$\gamma_s = \frac{E_s}{N_0}. \tag{8}$$

It is worth noting this performance will not be dependent on M since the both transmitters and receivers have equal number of antennas $(M+1)$.

B. Numerical Simulations

We performed Monte-Carlo simulations of perfect IA over the $M \times 2$ X-channel as explained in the previous section, where the transmit precoding vectors are normalized.Using simulations, we confirm that, with perfect global CSI knowledge, the BER performance of perfect IA in the $M \times 2$ X channel is exactly equal to that of the single user system with zero-forcing detectors. We also confirm that the simulated performance, labeled 'Sim. e=0.00', of the $M \times 2$ system is closely approximated with the analytical results of (7) and (8), labeled 'Th. e=0.00', and both are shown in Fig. 2, Fig. 4 and Fig. 5 for BPSK, QPSK and 16-PSK modulations, respectively. Due to perfect IA with ZF and equal number of transmit and receive antennas per terminal, the performance is independent on the number of users M or the number of transmit antennas. This agrees with the analysis in the previous sub-section and equations (7) and (8).

IV. Imperfect CSI at the Receiver Only

We analyze the case when the 2 receivers have imperfect CSI while the M transmitters have perfect CSI. This can be the case in Time Division Duplex (TDD) MIMO systems with perfect channel reciprocity. We assume that receivers have prior perfect knowledge of the alignment directions I_j. The NMSE of estimating each channel matrix at the receiver is e_r^2, where the CE error model of (6) is assumed. Whereas the transmit channel precoding matrices are exact, the detection matrix of receiver Rx$_j$ is estimated to be $\widehat{B}_j$.

A. Analytical Performance

The output of the ZF detector of Rx$_j$ can be modeled as

$$\widehat{\mathbf{z_j}} = \widehat{B}_j^{-1}\mathbf{y_j} = \widehat{B}_j^{-1}B_j\mathbf{x_j} + \widehat{B}_j^{-1}\mathbf{n_j}, \tag{9}$$

where from (3)

$$\widehat{B}_j = [\widehat{B}_{j,1} \ \cdots \ \widehat{B}_{j,M} \ I_j]. \tag{10}$$

Let $\mathbf{I}$ be the $(M+1)$ identity matrix, then using the linear Taylor approximation,

$$\widehat{H}_{mk}^{-1} = (H_{mk} + e_r\Omega_{mk})^{-1} \approx H_{mk}^{-1}\left(\mathbf{I} - e_r\Omega_{mk}H_{mk}^{-1}\right).$$

The mth column of $\widehat{B}_j$, $\widehat{B}_{j,m}$, can be written as

$$\begin{aligned} \widehat{B}_{j,m} &= \widehat{H}_{mj}\widehat{H}_{mk}^{-1}I_k \\ &= (H_{mj} + e_r\Omega_{mj})(H_{mk} + e_r\Omega_{mk})^{-1} I_k \\ &\approx H_{mj}H_{mk}^{-1}I_k + e_r\left(\Omega_{mj} - H_{mj}H_{mk}^{-1}\Omega_{mk}\right)I_k' \\ &\quad - e_r^2\Omega_{mj}H_{mk}^{-1}\Omega_{mk}I_k' \\ &\approx B_{j,m} + e_r\Omega_{j,m}', \end{aligned} \tag{11}$$

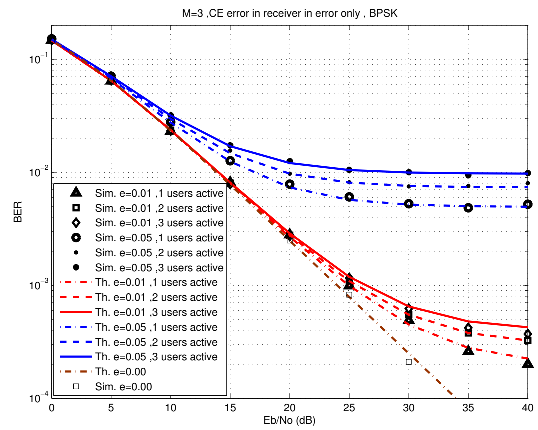

Fig. 2. Analytical and simulated BER performance of 3×2 IA, BPSK modulation, with receiver CSI error

for $k = \text{mod}(j,2) + 1$, where $I_k' = H_{mk}^{-1}I_k$. The terms with e_r^2 were neglected, as the CE NMSE is assumed to be small such that $e_r^2 \lll 1$. The elements of the column vector $\Omega_{j,m}'$ are of zero mean and variance 2. This can be intuitively understood by observing that the error variance affecting $\widehat{B}_{j,m}$ is contributed by the error variance in both $\widehat{H}_{mj}$ and $\widehat{H}_{mk}$. Consequently, the detection matrix at Rx$_j$ can be approximated as

$$\widehat{B}_j \approx B_j + e_r\Omega_j', \tag{12}$$

such that the elements of Ω_j' have zero mean and

$$\mathrm{E}[\Omega_j'(\Omega_j')^H] = 2(M+1)\mathbf{I}. \tag{13}$$

Using the linear Taylor approximation, $\widehat{B}_j^{-1} \approx B_j^{-1}\left(\mathbf{I} - e_r\Omega_j'B_j^{-1}\right)$. From (9), the ZF estimate at Rx$_j$ is

$$\widehat{\mathbf{z_j}} = \mathbf{x_j} + \widehat{\mathbf{n}}_\mathbf{j}, \tag{14}$$

where $\widehat{\mathbf{n}}_\mathbf{j}$ is the effective noise at the jth receiver, composed of the ZF enhanced noise plus the interference with other users' symbols due to imperfect CE, and is given by

$$\widehat{\mathbf{n}}_\mathbf{j} = B_j^{-1}\mathbf{n_j} - e_rB_j^{-1}\Omega_j'\mathbf{x_j} - e_rB_j^{-1}\Omega_j'B_j^{-1}\mathbf{n_j}. \tag{15}$$

In this case, the effective symbol SNR gain (*cf.* (7)) is

$$\gamma_s = \frac{E_s}{\mathrm{E}[\widehat{\mathbf{n}}_\mathbf{j}\widehat{\mathbf{n}}_\mathbf{j}^H]_{i,i}}, \tag{16}$$

for $i = 1, 2, \cdots, M$, where $X_{i,i}$ is the ith diagonal element of X. The $(M+1)$th dimension is not accounted for when calculating the effective SNR gain since it only carries the already aligned interference symbols. Following an analysis similar to the simpler case of single user MIMO ZF detection, the effective SNR gain of x_{ij} given in (16) can be approximated as

$$\gamma_s \approx \frac{E_s}{\mathrm{E}[\mathbf{n_j}\mathbf{n_j}^H]_{i,i} + e_r^2\,\mathrm{E}[(\Omega_j'\mathbf{x_j})(\Omega_j'\mathbf{x_j})^H]_{i,i}}, \tag{17}$$

where $e_r^2\,\text{trace}\left((B_j^HB_j)^{-1}\right)$ was neglected due to its small impact. If only Tx$_i$ is transmitting data, then from (13)

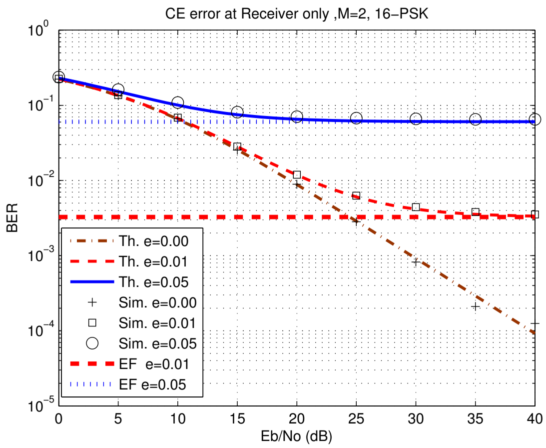

Fig. 3. Analytical and simulated BER performance for 2×2 IA, 16-PSK modulation, with receiver CSI error

$\mathrm{E}[(\Omega_j' \mathbf{x_j})(\Omega_j' \mathbf{x_j})^H]_{i,i} = \mathrm{E}[x_{ij} x_{ij}^H] \mathrm{E}[(\Omega_j')(\Omega_j')^H]_{i,i} = 2(M+1)E_s$. Generally, when *only* V *out of* M transmitters are sending data, $\mathrm{E}[(\Omega \mathbf{x_j})(\Omega \mathbf{x_j})^H]_{i,i} = (V+1)(M+1)E_s$, then

$$\gamma_s \approx \frac{E_s/N_0}{1 + e_r^2 (V+1)(M+1) E_s/N_0} \tag{18}$$

$$= \left. \frac{E_s/N_0}{1 + e_r^2 (M+1)^2 E_s/N_0} \right|_{V=M}. \tag{19}$$

Thus, the performance of $M \times 2$ IA with receiver CE error of NMSE e_r^2 is given by (7) for K-PSK modulation, where γ_s is respectively given by (18) or (19) if V or all M transmitters are active. We observe that: 1) The BER is dependent on $N_t = (M+1)$, as due to CSI error, ZF is not able to perfectly cancel the interference between the streams sent by the $(M+1)$ antennas of Tx_i. 2) The BER is dependent on $V+1$, since with CSI error, there is residual interference between the streams sent from the V users to Rx_j as well as from the other receivers' symbols aligned at the $(M+1)$th dimension. As the bit SNR tends to infinity, there is an error floor as the effective SNR is negatively dominated by inter-stream interference

$$\gamma_s^{\infty} \approx \frac{1}{e_r^2 (V+1)(M+1)}. \tag{20}$$

B. Numerical Simulations

In this section, we confirm the analytical BER expressions (7),(18), (19) and using Monte Carlo simulations of the $M \times 2$ X-channel IA scheme, with PSK modulations. We assume that the NMSE of estimating a channel matrix at any receiver is e_r^2. The rest of the simulation setup is same as that of subsection III.B. The equations were verified for different values of K, M, V and e_r and we found that they provide very accurate estimation of the simulated performance. For example, in Fig. 2, we show the performance in a 3×2 system with BPSK modulation, when different number of transmitters V are active. We note that ZF IA is very sensitive to CE errors. With a slight NMSE of $e_r = 0.01$, the performance degrades significantly compared to the case of

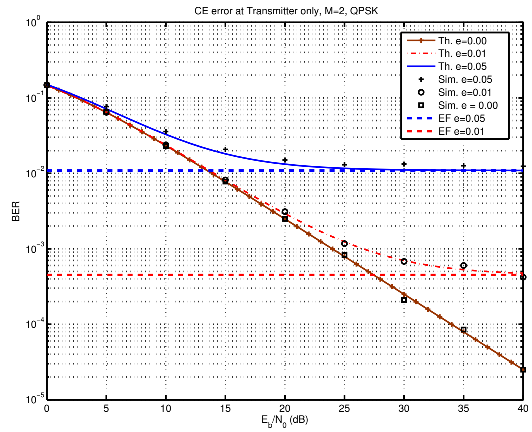

Fig. 4. Analytical and simulated BER performance of 2×2 IA, QPSK modulation, with transmitter CSI error

perfect CE and the there is an error floor at a BER of 10^{-3}. If the NMSE is larger, e.g. at $e_r = 0.05$, the system is useless due to the error floor dominated by interference. We also confirm the validity of the equations for 16-PSK with M=2 in Fig. 3, where the BER performance and error floors are shown at various receiver channel estimation qualities.

V. Imperfect CSI at the Transmitter Only

In this section, we consider the case when the transmitters have imperfect CSI knowledge but perfect prior knowledge of the aligning directions I_1 and I_2. The receivers are assumed to have perfect CE. This is a practical scenario due to CSI feedback errors and delays.

A. Analytical Performance

Assuming the transmitters estimate each channel with CE NMSE e_t^2, then the output of the ZF IA detector at Rx_j is

$$\widehat{\mathbf{z_j}} = B_j^{-1} \widehat{B}_j \mathbf{x_j} + B_j^{-1} \mathbf{n_j} = \mathbf{x_j} + \widehat{\mathbf{n_j}}, \tag{21}$$

where $\widehat{B}_j$ is given by (10) which can be approximated similar to (12) and (13). Then, the enhanced noise due to ZF and CE error is approximated as

$$\widehat{\mathbf{n_j}} \approx B_j^{-1} \mathbf{n_j} + e_t B_j^{-1} \Omega_j' \mathbf{x_j},. \tag{22}$$

Thus, the effective noise covariance matrix can be expressed as

$$\mathrm{E}[\widehat{\mathbf{n_j}} \widehat{\mathbf{n_j}}^H] \approx \left(N_0 + e_t^2 \mathrm{E}[\Omega_j' {\Omega_j'}^H] E_s \right) (B_j^H B_j)^{-1}$$

and the effective symbol SNR gain in case all the M transmitters are active can be expressed as

$$\gamma_s \approx \frac{E_s/N_0}{1 + e_t^2 (M+1)^2 E_s/N_0}. \tag{23}$$

At infinite bit SNR, the error floor due to inter-stream interference is given by

$$\gamma_s^{\infty} \approx \frac{1}{e_t^2 (M+1)^2}. \tag{24}$$

The corresponding BER in case of PSK modulation is calculated by substituting (23) or (24) in (7).

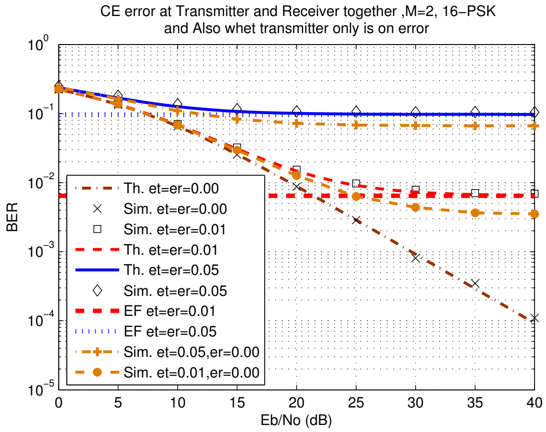

Fig. 5. Analytical and simulated BER performance of 2×2 IA, 16-PSK modulation, with transmitter and receiver CSI errors

B. Numerical Simulations

Using simulations, we validate the analytical BER performance, calculated by (23), (24) and (7) in case each transmitter has independent CE for each channel matrix with NMSE e_t^2. This is demonstrated in Fig. 4 for the 2×2 X-channel with QPSK modulation for $e_t = 0, 0.01$ and 0.05.

VI. Imperfect CSI at all Receivers and Transmitters

In this section, we analyze the BER performance of the more practical scenario when each channel matrix is estimated at each receiver and transmitter independently with CE NMSE e_r^2 and e_t^2, respectively.

A. Analytical Performance

The transmitters and receiver Rx_j will independently estimate B_j, respectively as $\widehat{B}_j \approx B_j + e_r\Omega'_j$ and $\widetilde{B}_j \approx B_j + e_t\Omega''_j$, where Ω'_j and Ω''_j are independent and the approximation is as shown by (11) and (12). The effective noise can be expressed as

$$\begin{aligned}\widehat{\mathbf{n}_\mathbf{j}} \approx B_j^{-1}\mathbf{n_j} - e_r B_j^{-1}\Omega'_j\mathbf{x_j} - e_r B_j^{-1}\Omega'_j B_j^{-1}\mathbf{n_j} \\ + e_t B_j^{-1}\Omega''_j\mathbf{x_j} - e_r e_t B_j^{-1}\Omega'_j B_j^{-1}\Omega''_j\mathbf{x_j}.\end{aligned} \tag{25}$$

When V transmitters are active, the effective SNR gain is

$$\gamma_s \approx \frac{E_s/N_0}{1 + (e_r^2 + e_t^2)(V+1)(M+1)E_s/N_0}, \tag{26}$$

and the error floor due to residual inteference after ZF is

$$\gamma_s^\infty \approx \frac{1}{(e_r^2 + e_t^2)(V+1)(M+1)}. \tag{27}$$

B. Simulated Performance

Fig. 5 shows the BER performance of IA in the 2×2 X-channel, with 16-PSK modulation, and both transmitters are active. We verify that the analytical equations of (eq:TxRxEF), (26) and (7) provide a tight approximation to the simulated performance. By comparing to the case of CE at the transmitter only, we quantify the effect of having imperfect CSI feedback.

VII. Conclusions

We analyzed the error rate performance of perfect interference alignment (IA) in $M \times 2$ MIMO X channels with zero-forcing detectors at finite signal to noise ratios. Our analytical expressions were shown to closely approximate the simulated performance. We show that in case of perfect global channel knowledge at the transmitters and receivers, the performance is the same as that of a MIMO single user system. We observe that with perfect IA, the degradation in performance due to imperfect channel knowledge is similar in case the estimation error is at the transmitter only or at the receiver only. We observe that the performance of IA is very sensitive to slight channel estimation errors and severe error floors exist due to residual interference. Thus, more robust IA techniques should be investigated.

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